Matrix

An $m \times n$ (read "m by n") ***matrix***, denoted by **A**, is a rectangular array of entries or elements (numbers, or symbols representing numbers) enclosed typically by square brackets, where m is the number of rows and n the number of columns.

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

A gray-scale image is often represented by a matrix whose elements are intensity values of pixels

Matrix Definitions (Con't)

- **A** is ***square*** if $m = n$.
- **A** is ***diagonal*** if all off-diagonal elements are 0, and not all diagonal elements are 0.
- **A** is the ***identity matrix*** (**I**) if it is diagonal and all diagonal elements are 1.
- **A** is the ***zero*** or ***null matrix*** (**0**) if all its elements are 0.
- The ***trace*** of **A** equals the sum of the elements along its main diagonal.
- Two matrices **A** and **B** are ***equal*** iff the have the same number of rows and columns, and $a_{ij} = b_{ij}$.

Matrix Definitions (Con't)

- The ***transpose*** $\mathbf{A}^T$ of an $m \times n$ matrix **A** is an $n \times m$ matrix obtained by interchanging the rows and columns of **A**.
- A square matrix for which $\mathbf{A}^T = \mathbf{A}$ is said to be ***symmetric***.
- Any matrix X for which **XA**=**I** and **AX**=**I** is called the ***inverse*** of **A**.
- Let c be a real or complex number (called a ***scalar***). The ***scalar multiple*** of c and matrix **A**, denoted $c\mathbf{A}$, is obtained by multiplying every elements of **A** by c. If $c = -1$, the scalar multiple is called the ***negative*** of **A**.

Block Matrix

A ***block matrix*** is a matrix that is defined using smaller matrices, called blocks

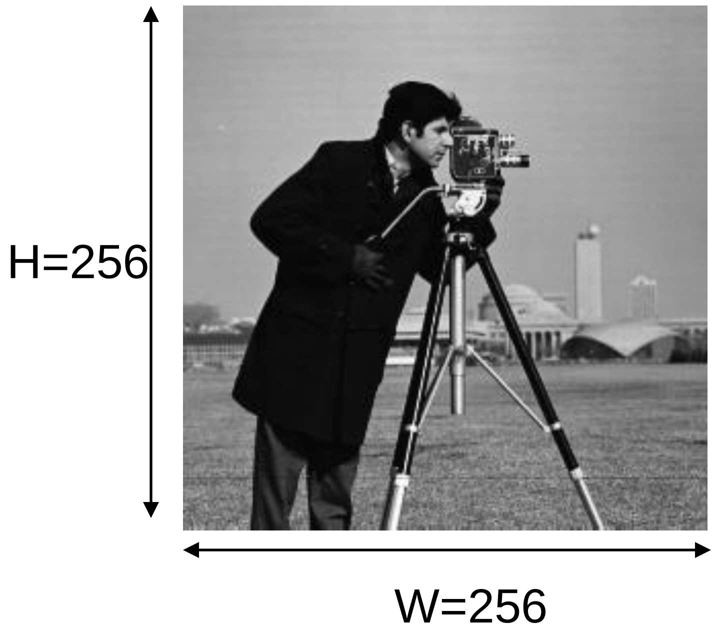
Example $$A=\begin{bmatrix}1 & 2\\ 3 & 4\end{bmatrix}, B=\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}, C=\begin{bmatrix}3 & 4\\ 5 & 6\end{bmatrix}, D=\begin{bmatrix}4 & 5\\ 6 & 7\end{bmatrix}$$

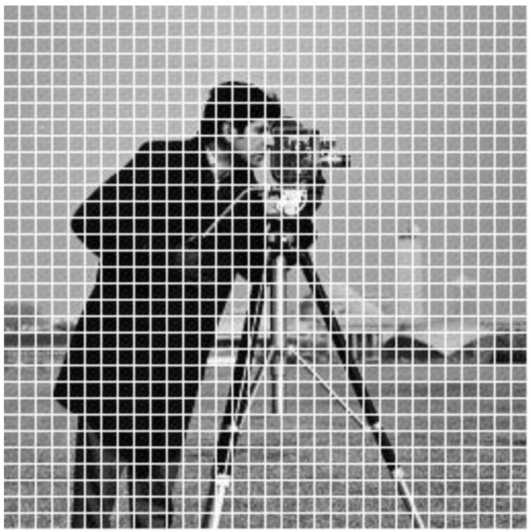
$$\Rightarrow \quad \begin{bmatrix}A & B\\ C & D\end{bmatrix}=\left[\begin{array}{cc:cc}1 & 2 & 2 & 3\\ 3 & 4 & 4 & 5\\ \hdashline 3 & 4 & 4 & 5\\ 5 & 6 & 6 & 7\end{array}\right]$$

Exercise (Hadamard Matrix)

$$A_1=\begin{bmatrix}1 & 1\\ 1 & -1\end{bmatrix}, A_{2n}=\begin{bmatrix}A_n & A_n\\ A_n & -A_n\end{bmatrix}$$

Block Matrix (con't)





divided into 1024 8×8 block matrices in JPEG compression

Row and Column Vectors

A ***column vector*** is an $m \times 1$ matrix:

$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix}$$

A ***row vector*** is a $1 \times n$ matrix:

$$\boxed{\mathbf{b} = [b_1, b_2, \cdots b_n]}$$

A column vector can be expressed as a row vector by using the transpose:

$$\boxed{\mathbf{a}^T = [a_1, a_2, \cdots, a_m]}$$

Vector Norms

There are numerous norms that are used in practice. In our work, the norm most often used is the so-called ***2-norm***, which, for a vector **x** in real $\Re^m$, space is defined as

$$\boxed{\|\mathbf{x}\| = \left[x_1^2 + x_2^2 + \cdots + x_m^2\right]^{1/2}}$$

which is recognized as the *Euclidean distance* from the origin to point **x**; this gives the expression the familiar name Euclidean norm. The expression also is recognized as the length of a vector **x**, with origin at point **0**. From earlier discussions, the norm also can be written as

$$\boxed{\|\mathbf{x}\| = \left[\mathbf{x}^T\mathbf{x}\right]^{1/2}}$$

Some Basic Matrix Operations

- The ***sum*** of two matrices **A** and **B** (of equal dimension), denoted **A** + **B**, is the matrix with elements $a_{ij} + b_{ij}$.
- The ***difference*** of two matrices, **A**– **B**, has elements $a_{ij} - b_{ij}$.
- The ***product***, **AB**, of $m \times n$ matrix **A** and $n \times q$ matrix **B**, is an $m \times q$ matrix **C** whose (i,j)-th element is formed by multiplying the entries across the ith row of **A** times the entries down the jth column of **B**; that is,

$$\boxed{c_{ij} = \sum_{k=1}^{n} a_{ik} b_{kj}}$$

Some Basic Matrix Operations (Con't)

The ***inner product*** (also called ***dot product***) of two vectors

$$\mathbf{a} = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{bmatrix} \qquad \mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix}$$

is defined as

$$\boxed{\begin{aligned} \mathbf{a}^T\mathbf{b} = \mathbf{b}^T\mathbf{a} &= a_1b_1 + a_2b_2 + \cdots + a_mb_m \\ &= \sum_{i=1}^{m} a_ib_i. \end{aligned}}$$

Note that the inner product is a scalar.

Inner Product

The ***Cauchy-Schwartz*** inequality states that

$$\boxed{|\mathbf{x}^T\mathbf{y}| \leq \|\mathbf{x}\|\,\|\mathbf{y}\|}$$

Another well-known result used in the book is the expression

$$\boxed{\cos\theta = \frac{\mathbf{x}^T\mathbf{y}}{\|\mathbf{x}\|\,\|\mathbf{y}\|}}$$

where θ is the angle between vectors **x** and **y**. From these expressions it follows that the inner product of two vectors can be written as

$$\boxed{\mathbf{x}^T\mathbf{y} = \|\mathbf{x}\|\,\|\mathbf{y}\|\cos\theta}$$

Thus, the inner product can be expressed as a function of the norms of the vectors and the angle between the vectors.

Geometric Intuition

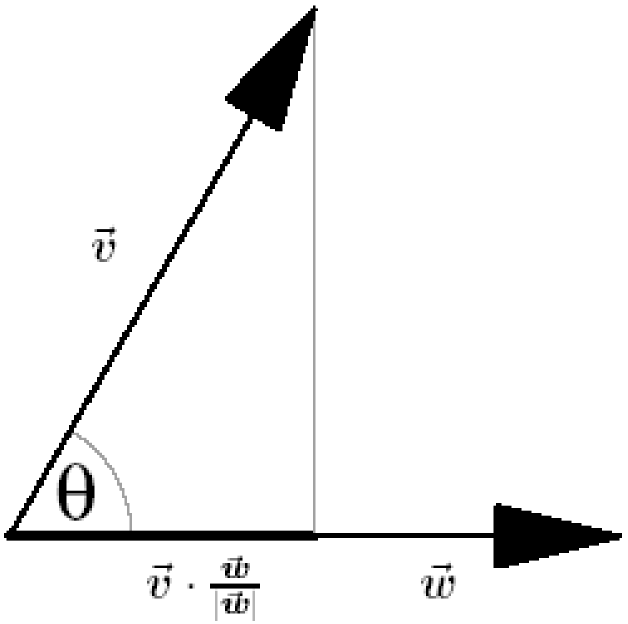


Figure 1: The dot product is fundamentally a projection.

$$\vec{v} \cdot \frac{\vec{w}}{|\vec{w}|} = |\vec{v}|\cos\theta \Rightarrow \vec{v} \cdot \vec{w} = |\vec{v}||\vec{w}|\cos\theta$$

Orthogonality and Orthonormality

From the preceding results, two vectors in $\Re^m$ are ***orthogonal*** if and only if their inner product is zero. Two vectors are ***orthonormal*** if, in addition to being orthogonal, the length of each vector is 1.

From the concepts just discussed, we see that an arbitrary vector $\mathbf{a}$ is turned into a vector $\mathbf{a}_n$ of unit length by performing the operation $\mathbf{a}_n = \mathbf{a}/\|\mathbf{a}\|$. Clearly, then, $\|\mathbf{a}_n\| = 1$.

A ***set of vectors*** is said to be an ***orthogonal*** set if every two vectors in the set are orthogonal. A ***set of vectors*** is ***orthonormal*** if every two vectors in the set are orthonormal.

Some Important Aspects of Orthogonality

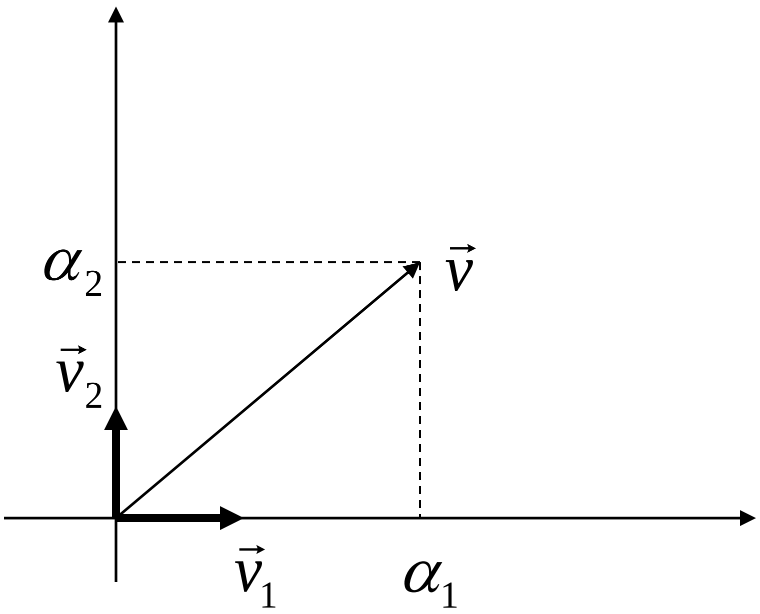
Let $B = \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ be an orthogonal or orthonormal basis in the sense defined in the previous section. Then, an important result in vector analysis is that any vector v can be represented with respect to the orthogonal basis B as

$$\boxed{\mathbf{v} = \alpha_1 \mathbf{v}_1 + \alpha_2 \mathbf{v}_2 + \cdots + \alpha_n \mathbf{v}_n}$$

where the coefficients are given by

$$\boxed{\begin{aligned} \alpha_i &= \frac{\mathbf{v}^T \mathbf{v}_i}{\mathbf{v}_i^T \mathbf{v}_i} \\ &= \frac{\mathbf{v}^T \mathbf{v}_i}{\|\mathbf{v}_i\|^2} \end{aligned}}$$

Geometric Example



$$\vec{v} = \alpha_1 \vec{v}_1 + \alpha_2 \vec{v}_2$$

Sets and Set Operations

Probability events are modeled as sets, so it is customary to begin a study of probability by defining sets and some simple operations among sets.

A ***set*** is a collection of objects, with each object in a set often referred to as an ***element*** or ***member*** of the set. Familiar examples include the set of all image processing books in the world, the set of prime numbers, and the set of planets circling the sun. Typically, sets are represented by uppercase letters, such as A, B, and C, and members of sets by lowercase letters, such as a, b, and c.

Sets and Set Operations (Con't)

We denote the fact that an ***element a belongs*** to set A by

$$\boxed{a \in A}$$

If a is not an element of A, then we write

$$\boxed{a \notin A}$$

A set can be specified by listing all of its elements, or by listing properties common to all elements. For example, suppose that I is the set of all integers. A set B consisting the first five nonzero integers is specified using the notation

$$\boxed{B = \{1, 2, 3, 4, 5\}}$$

Sets and Set Operations (Con't)

The set of all integers less than 10 is specified using the notation

$$\boxed{C = \{c \in I \mid c < 10\}}$$

which we read as "*C* is the set of integers such that each members of the set is less than 10." The "such that" condition is denoted by the symbol " | " . As shown in the previous two equations, the elements of the set are enclosed by curly brackets.

The set with no elements is called the ***empty*** or ***null set***, denoted in this review by the symbol Ø.

Sets and Set Operations (Con't)

Two sets A and B are said to be ***equal*** if and only if they contain the same elements. Set equality is denoted by

$$\boxed{A = B}$$

If the elements of two sets are not the same, we say that the sets are ***not equal***, and denote this by

$$\boxed{A \neq B}$$

If every element of B is also an element of A, we say that B is a ***subset*** of A:

$$\boxed{B \subseteq A}$$

Some Basic Set Operations

The operations on sets associated with basic probability theory are straightforward. The ***union*** of two sets *A* and *B*, denoted by

$$\boxed{A \cup B}$$

is the set of elements that are either in *A* or in *B*, or in both. In other words,

$$\boxed{A \cup B = \{z \mid z \in A \text{ or } z \in B\}}$$

Similarly, the ***intersection*** of sets *A* and *B*, denoted by

$$\boxed{A \cap B}$$

is the set of elements common to both *A* and *B*; that is,

$$\boxed{A \cap B = \{z \mid z \in A \text{ and } z \in B\}}$$

Set Operations (Con't)

Two sets having no elements in common are said to be ***disjoint*** or ***mutually exclusive***, in which case

$$\boxed{A \cap B = \emptyset}$$

The ***complement*** of set *A* is defined as

$$\boxed{A^c = \{z | z \notin A\}}$$

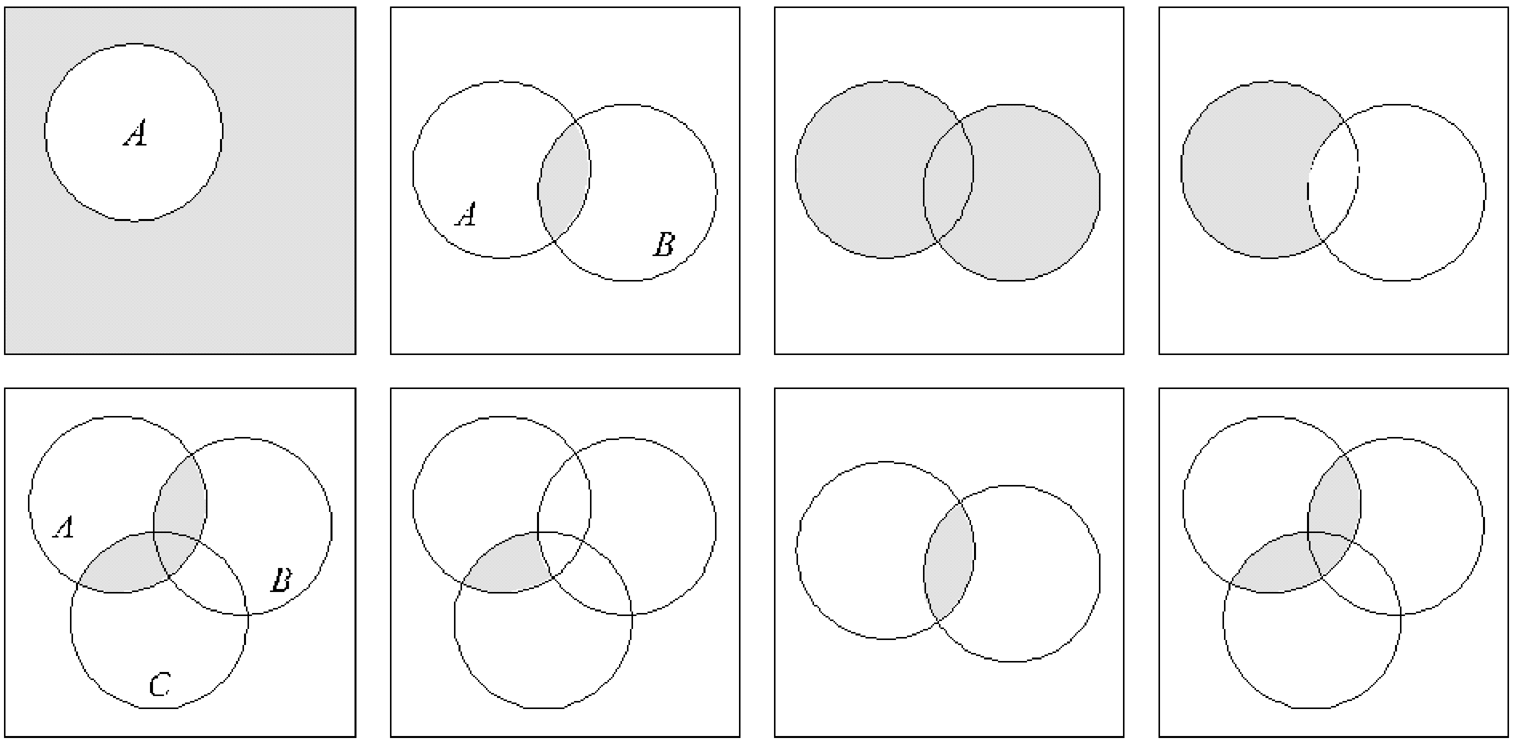
Clearly, $(A^c)^c = A$. Sometimes the complement of *A* is denoted as $\overline{A}$.

The ***difference*** of two sets *A* and *B*, denoted $A - B$, is the set of elements that belong to *A*, but not to *B*. In other words,

$$\boxed{A - B = \{z | z \in A, \ z \notin B\}}$$

Set Operations (Con't)

A^c $\quad$ $A \cap B$ $\quad$ $A \cup B$ $\quad$ $A - B$



$A \cap (B \cup C)$ $\quad$ $(A \cap C) - (A \cap B \cap C)$ $\quad$ $A \cap B$ $\quad$ $(A \cap B) \cup (A \cap C) - (A \cap B \cap C)$

Relative Frequency & Probability

A ***random experiment*** is an experiment in which it is not possible to predict the outcome. Perhaps the best known random experiment is the tossing of a coin. Assuming that the coin is not biased, we are used to the concept that, on average, half the tosses will produce heads (H) and the others will produce tails (T). Let n denote the total number of tosses, n_H the number of heads that turn up, and n_T the number of tails. Clearly,

$$\boxed{n_H + n_T = n.}$$

Relative Frequency & Prob. (Con't)

Dividing both sides by *n* gives

$$\boxed{\frac{n_H}{n} + \frac{n_T}{n} = 1.}$$

The term n_H/n is called the ***relative frequency*** of the event we have denoted by *H*, and similarly for n_T/n. If we performed the tossing experiment a large number of times, we would find that each of these relative frequencies tends toward a stable, limiting value. We call this value the ***probability of the event***, and denoted it by *P*(event).

Relative Frequency & Prob. (Con't)

In the current discussion the probabilities of interest are $P(H)$ and $P(T)$. We know in this case that $P(H) = P(T) = 1/2$. Note that the event of an experiment need not signify a single outcome. For example, in the tossing experiment we could let D denote the event "heads or tails," (note that the event is now a set) and the event E, "neither heads nor tails." Then, $P(D) = 1$ and $P(E) = 0$.

The first important property of P is that, for an event A,

$$\boxed{0 \leq P(A) \leq 1.}$$

That is, the probability of an event is a positive number bounded by 0 and 1. Let S be the set of all possible events

$$\boxed{P(S) = 1.}$$

Random Variables

Random variables often are a source of confusion when first encountered. This need not be so, as the concept of a random variable is in principle quite simple. A ***random variable***, *x*, is a ***real-valued function*** defined on the events of the sample space, *S*. In words, for each event in *S*, there is a real number that is the corresponding value of the random variable. Viewed yet another way, a random variable maps each event in *S* onto the real line. That is it. A simple, straightforward definition.

Discrete: denumerable events
Continuous: indenumerable events

Discrete Random Variables

Example I: Consider again the experiment of drawing a single card from a standard deck of 52 cards. Suppose that we define the following events. A: a heart; B: a spade; C: a club; and D: a diamond, so that $S = \{A, B, C, D\}$. A random variable is easily defined by letting $x = 1$ represent event A, $x = 2$ represent event B, and so on.

event	notation	probability
A	$x = 1$	1/4
B	$x = 2$	1/4
C	$x = 3$	1/4
D	$x = 4$	1/4

Discrete Random Variables (Con't)

Example II:, consider the experiment of throwing a single die with two faces of "1" and no face of "6". Let us use $x=1,2,3,4,5$ to denote the five possible events. Then the random variable X is defined by

event	notation	probability
"1" appears	$x = 1$	1/3
"2" appears	$x = 2$	1/6
"3" appears	$x = 3$	1/6
"4" appears	$x = 4$	1/6
"5" appears	$x = 5$	1/6

Therefore, the probability of getting "1,2,1" in the expepriment of throwing the dice three times is $1/3\times1/6\times1/3=1/54$

Discrete Random Variables (Con't)

Example III: For a gray-scale image ($L=256$), we can use the notation $p(r_k)$, $k = 0,1,\ldots, L - 1$, to denote the ***histogram*** of an image with L possible gray levels, r_k, $k = 0,1,\ldots, L - 1$, where $p(r_k)$ is the probability of the kth gray level (random event) occurring. The discrete random variables in this case are gray levels.

Question: Given an image, how to calculate its histogram?

You will be asked to do this using MATLAB in the first computer assignment

Continuous Random Variables

Thus far we have been concerned with random variables whose values are discrete. To handle ***continuous random variables*** we need some additional tools. In the discrete case, the probabilities of events are numbers between 0 and 1. When dealing with continuous quantities (which are not denumerable) we can no longer talk about the "probability of an event" because that probability is zero. This is not as unfamiliar as it may seem. For example, given a continuous function we know that the area of the function between two limits a and b is the integral from a to b of the function. However, the area ***at a point*** is zero because the integral from,say, a to a is zero. We are dealing with the same concept in the case of continuous random variables.

Continuous Random Variables (Con't)

Thus, instead of talking about the probability of a specific value, we talk about the probability that the value of the random variable lies in a specified ***range***. In particular, we are interested in the probability that the random variable is less than or equal to (or, similarly, greater than or equal to) a specified constant a. We write this as

$$\boxed{F(a) = P(x \leq a).}$$

If this function is given for all values of a (i.e., $-\infty < a < \infty$), then the values of random variable x have been defined. Function F is called the ***cumulative probability distribution function*** or simply the ***cumulative distribution function*** (cdf).

Continuous Random Variables (Con't)

Due to the fact that it is a probability, the cdf has the following properties:

1. $F(-\infty) = 0$
2. $F(\infty) = 1$
3. $0 \leq F(x) \leq 1$
4. $F(x_1) \leq F(x_2) \quad \text{if} \quad x_1 < x_2$
5. $P(x_1 < x \leq x_2) = F(x_2) - F(x_1)$
6. $F(x^+) = F(x)$,

where $x^+ = x + \varepsilon$, with ε being a positive, infinitesimally small number.

Continuous Random Variables (Con't)

The ***probability density function*** (pdf) of random variable *x* is defined as the derivative of the cdf:

$$\boxed{p(x) = \frac{dF(x)}{dx}.}$$

The term ***density function*** is commonly used also. The pdf satisfies the following properties:

1. $p(x) \geq 0$ for all x
2. $\int_{-\infty}^{\infty} p(x)dx = 1$
3. $F(x) = \int_{-\infty}^{x} p(\alpha)d\alpha$, where α is a dummy variable
4. $P(x_1 < x \leq x_2) = \int_{x_1}^{x_2} p(x)dx.$

Mean of a Random Variable

The mean of a random variable X is defined by

$$\boxed{E[x] = \overline{x} = m = \int_{-\infty}^{\infty} xp(x)dx}$$

when x is continuos and

$$\boxed{E[x] = \overline{x} = m = \sum_{i=1}^{N} x_i P(x_i)}$$

when x is discrete.

$\overline{x}$

Variance of a Random Variable

The ***variance*** of a random variable, is defined by

$$\boxed{\sigma^2 = E[x^2] = \int_{-\infty}^{\infty} x^2 p(x) dx}$$

for continuous random variables and

$$\boxed{\sigma^2 = E[x^2] = \sum_{i=1}^{N} x_i^2 P(x_i)}$$

for discrete variables.

Normalized Variance & Standard Deviation

Of particular importance is the variance of random variables that have been ***normalized*** by subtracting their mean. In this case, the variance is

$$\boxed{\sigma^2 = E[(x-m)^2] = \int_{-\infty}^{\infty}(x-m)^2 p(x)dx}$$

and

$$\boxed{\sigma^2 = E[(x-m)^2] = \sum_{i=1}^{N}(x_i - m)^2 P(x_i)}$$

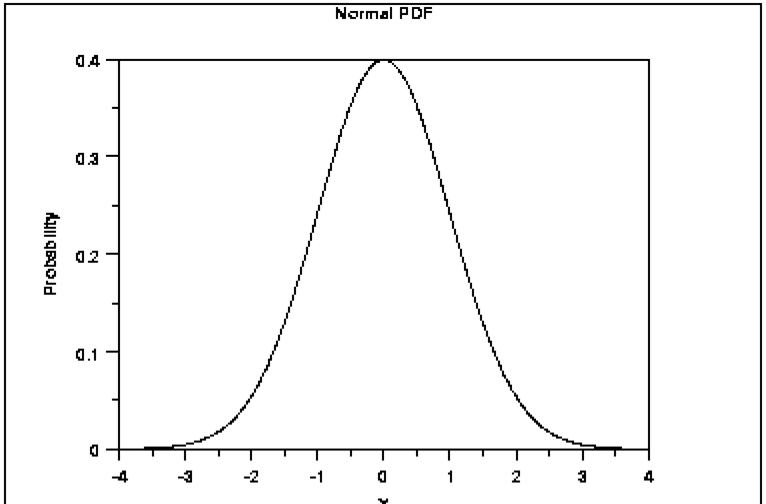
for continuous and discrete random variables, respectively. The square root of the variance is called the ***standard deviation***, and is denoted by σ.

The Gaussian Probability Density Function

A random variable is called ***Gaussian*** if it has a probability density of the form

$$p(x) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-m)^2/\sigma^2}$$

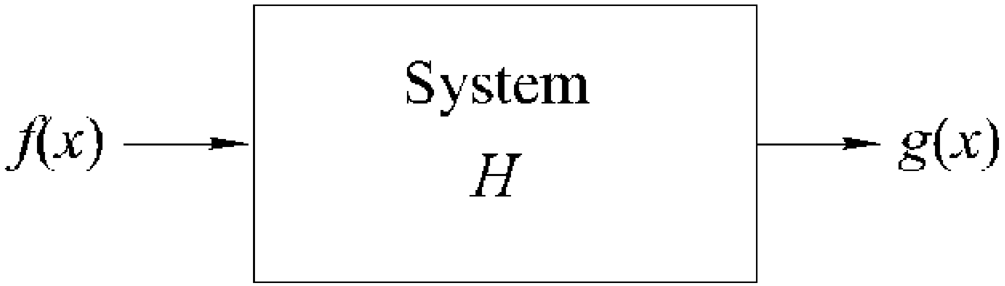
where *m* and σ are the mean and variance respectively. A Gaussin random variable is often denoted by N(m,σ^2)



pdf of N(0,1)

System

With reference to the following figure, we define a ***system*** as a unit that converts an input function $f(x)$ into an output (or response) function $g(x)$, where x is an independent variable, such as time or, as in the case of images, spatial position. We assume for simplicity that x is a continuous variable, but the results that will be derived are equally applicable to discrete variables.



System Operator

It is required that the system output be determined completely by the input, the system properties, and a set of initial conditions. From the figure in the previous page, we write

$$\boxed{g(x) = H[f(x)]}$$

where H is the ***system operator***, defined as a mapping or assignment of a member of the set of possible outputs $\{g(x)\}$ to each member of the set of possible inputs $\{f(x)\}$. In other words, the system operator completely characterizes the system response for a given set of inputs $\{f(x)\}$.

Linearity

An operator H is called a ***linear operator*** for a class of inputs $\{f(x)\}$ if

$$\boxed{\begin{aligned} H[a_i f_i(x) + a_j f_j(x)] &= a_i H[f_i(x)] + a_j H[f_{ji}(x)] \\ &= a_i g_i(x) + a_j g_j(x) \end{aligned}}$$

for all $f_i(x)$ and $f_j(x)$ belonging to $\{f(x)\}$, where the a's are arbitrary constants and

$$\boxed{g_i(x) = H[f_i(x)]}$$

is the output for an arbitrary input $f_i(x) \in \{f(x)\}$.

Time Invariance

An operator H is called ***time invariant*** (if x represents time), ***spatially invariant*** (if x is a spatial variable), or simply ***fixed parameter***, for some class of inputs $\{f(x)\}$ if

$$\boxed{g_i(x) = H[f_i(x)] \text{ implies that } g_i(x+x_0) = H[f_i(x+x_0)]}$$

for all $f_i(x) \in \{f(x)\}$ and for all x_0.

Discrete LTI System: digital filter

$$f(n) \longrightarrow \boxed{h(n)} \longrightarrow g(n)$$

$$g(n) = \sum_{k=-\infty}^{\infty} h(k) f(n-k) = h(n) \otimes f(n)$$

Linear convolution

- Linearity $\quad a_1 f_1(n) + a_2 f_2(n) \rightarrow a_1 g_1(n) + a_2 g_2(n)$

- Time-invariant property $\quad f(n-n_0) \rightarrow g(n-n_0)$

Communitive Property of Convolution

$$h(n) \otimes f(n) = \sum_{k=-\infty}^{\infty} h(k) f(n-k)$$

$$= \sum_{r=-\infty}^{\infty} h(n-r) f(r) = f(n) \otimes h(n)$$

$$r = n - k$$

<u>Conclusion</u>

The order of linear convolution doesn't matter.

Continuous Fourier Transform

forward $$F(w) = \int_{-\infty}^{\infty} f(x) e^{-j2\pi wx} dx$$

$$\updownarrow$$

inverse $$f(x) = \int_{-\infty}^{\infty} F(w) e^{j2\pi wx} dw$$

By representing a signal in frequency domain, FT facilitates the analysis and manipulation of the signal (e.g., think of the classification of female and male speech signals)

Fourier Transform of Sequences (Fourier Series)

forward

$$F(w) = \sum_{-\infty}^{\infty} f(n) e^{-jwn}$$

$$\updownarrow$$

inverse

$$f(n) = \frac{1}{2\pi} \int_{-\pi}^{\pi} F(w) e^{jwn} dw$$

time-domain convolution

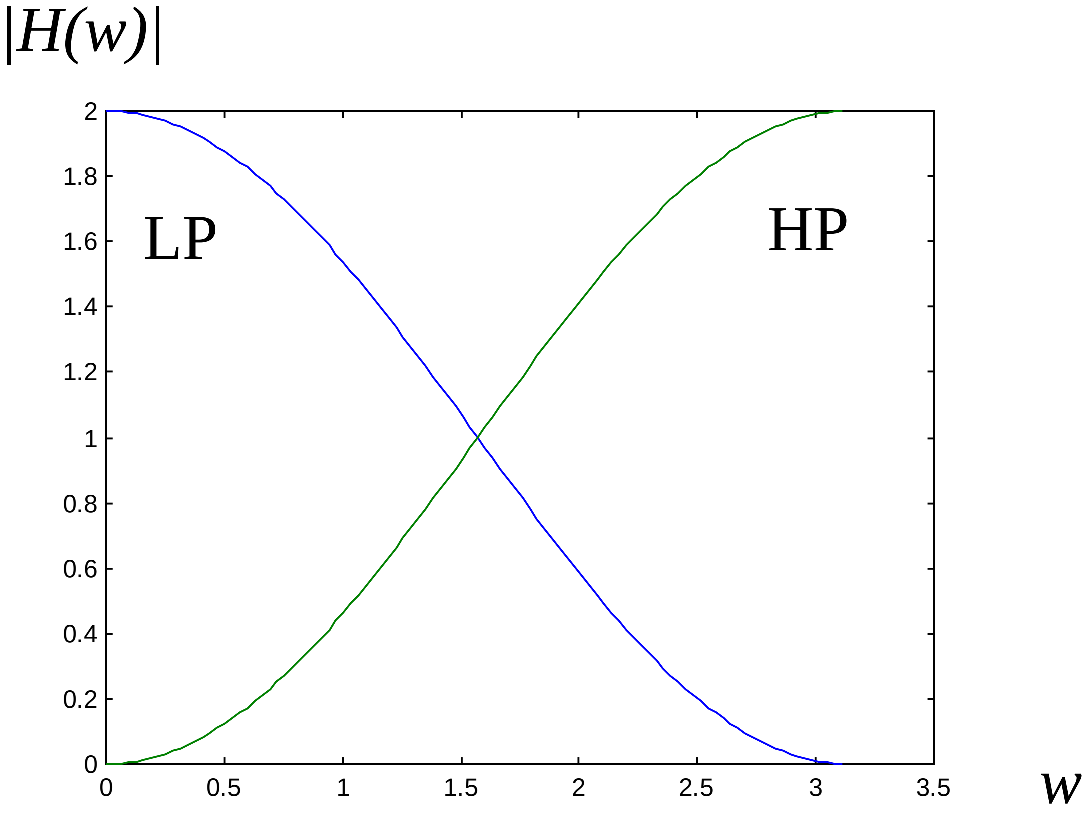
$$f(n) \otimes h(n)$$

frequency-domain multiplication

$$F(w) H(w)$$

Note that the input signal is a discrete sequence while its FT is a continuous function

- Low-pass and high-pass filters:



Examples: LP-h(n)=[1 1]/2   HP-h(n)=[1 -1]/2

- Frequency domain analysis

$$x(n) \underset{\text{IFT}}{\overset{\text{FT}}{\rightleftarrows}} X(w) = \sum_n x(n)e^{-jnw}$$

time frequency

- Properties of FT

-periodic $X(w+2k\pi) = X(w), \forall k \in Z$

-time shifting $x(n-n_0) \leftrightarrow e^{-jwn_0} X(w)$

-modulation $e^{-jw_0 n} x(n) \leftrightarrow X(w+w_0)$

-convolution $x(n) * h(n) \leftrightarrow X(w)H(w)$

Discrete Fourier Transform

forward
$$F(k)=\frac{1}{N}\sum_{n=0}^{N-1} f(n)W_N^{kn},\, k=0,1,\ldots,N-1$$

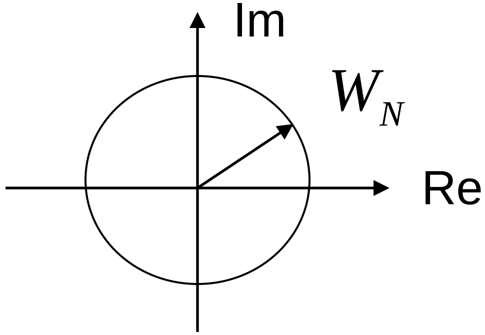
$$W_N=\exp\{-\frac{j2\pi}{N}\}$$

inverse
$$f(n)=\frac{1}{N}\sum_{n=0}^{N-1} F(k)W_N^{-kn},\, n=0,1,\ldots,N-1$$

<u>Exercise</u>

1. proof $W_N^N=1$

2. What is $\exp\{-j\pi\}$?



Two-dimensional Extension

- 2D filter

  -2D filter is defined as 2D convolution

$$y(m,n) = \sum_{k,l=-\infty}^{\infty} h(m-k, n-l)\, x(k,l)$$

  -we mostly consider 2D separable filters, i.e.

$$h(m,n) = h_r(m)\, h_c(n)$$

$$x(m,n) \longrightarrow \boxed{h(m,n)} \longrightarrow y(m,n)$$

$$\Updownarrow$$

$$x(m,n) \longrightarrow \boxed{h_r(m)} \longrightarrow \boxed{h_c(m)} \longrightarrow y(m,n)$$

- 2D Frequency domain analysis

$$x(m,n) \underset{\text{IFT}}{\overset{\text{FT}}{\rightleftarrows}} X(w_1, w_2) = \sum_{m,n} x(m,n) e^{-j(mw_1 + nw_2)}$$

time frequency

- Properties of 2D FT

-periodic $X(w_1 + 2k\pi, w_2 + 2l\pi) = X(w_1, w_2), \forall k, l \in Z$

-time shifting $x(m+m_0, n+n_0) \leftrightarrow e^{j(m_0 w_1 + n_0 w_2)} X(w_1, w_2)$

-modulation $e^{-j(w_{01}m + w_{02}n)} x(m,n) \leftrightarrow X(w_1 + w_{01}, w_2 + w_{02})$

-convolution $x(m,n) * h(m,n) \leftrightarrow X(w_1, w_2) H(w_1, w_2)$

Image Acquisition

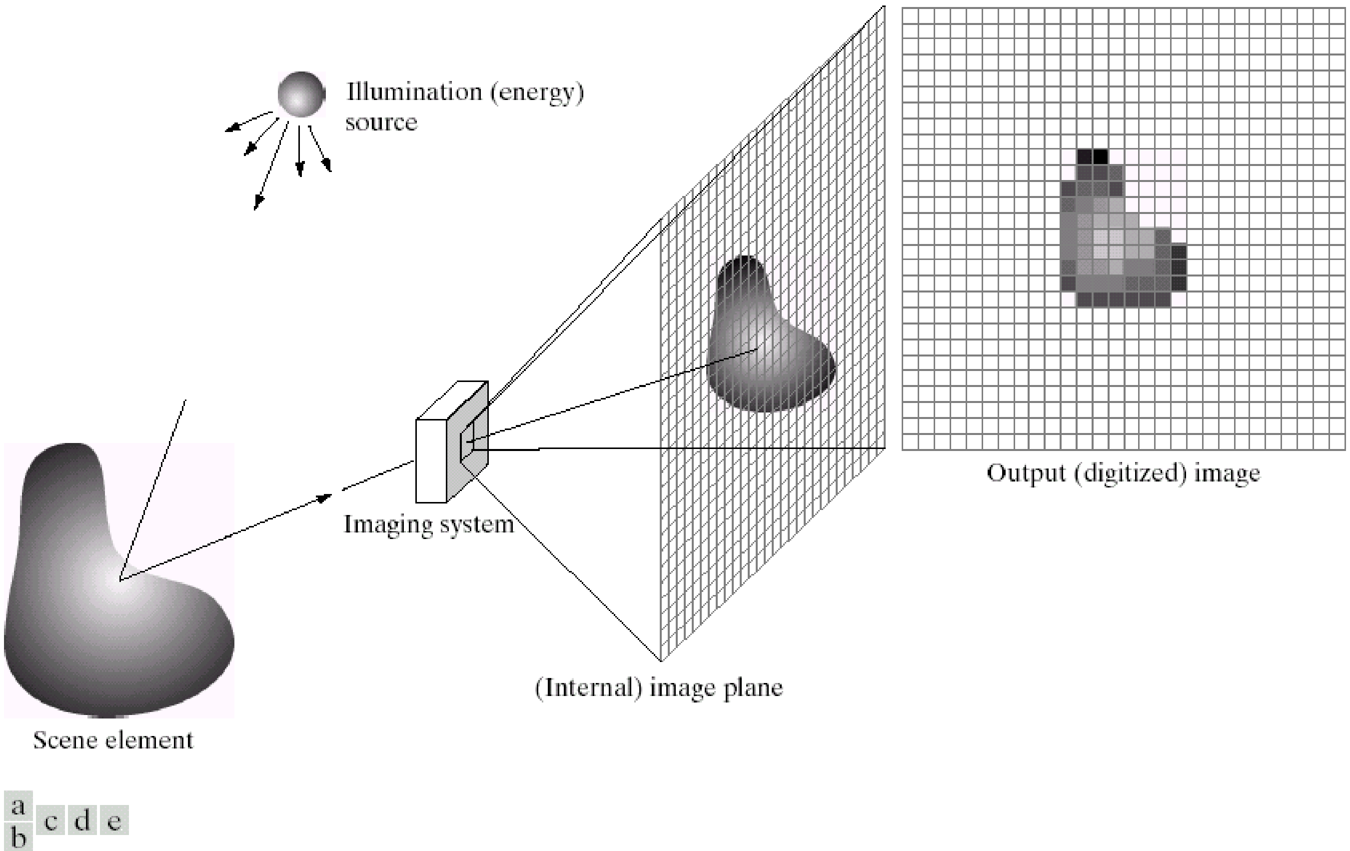


FIGURE 2.15 An example of the digital image acquisition process. (a) Energy ("illumination") source. (b) An element of a scene. (c) Imaging system. (d) Projection of the scene onto the image plane. (e) Digitized image.

Sampling and Quantization

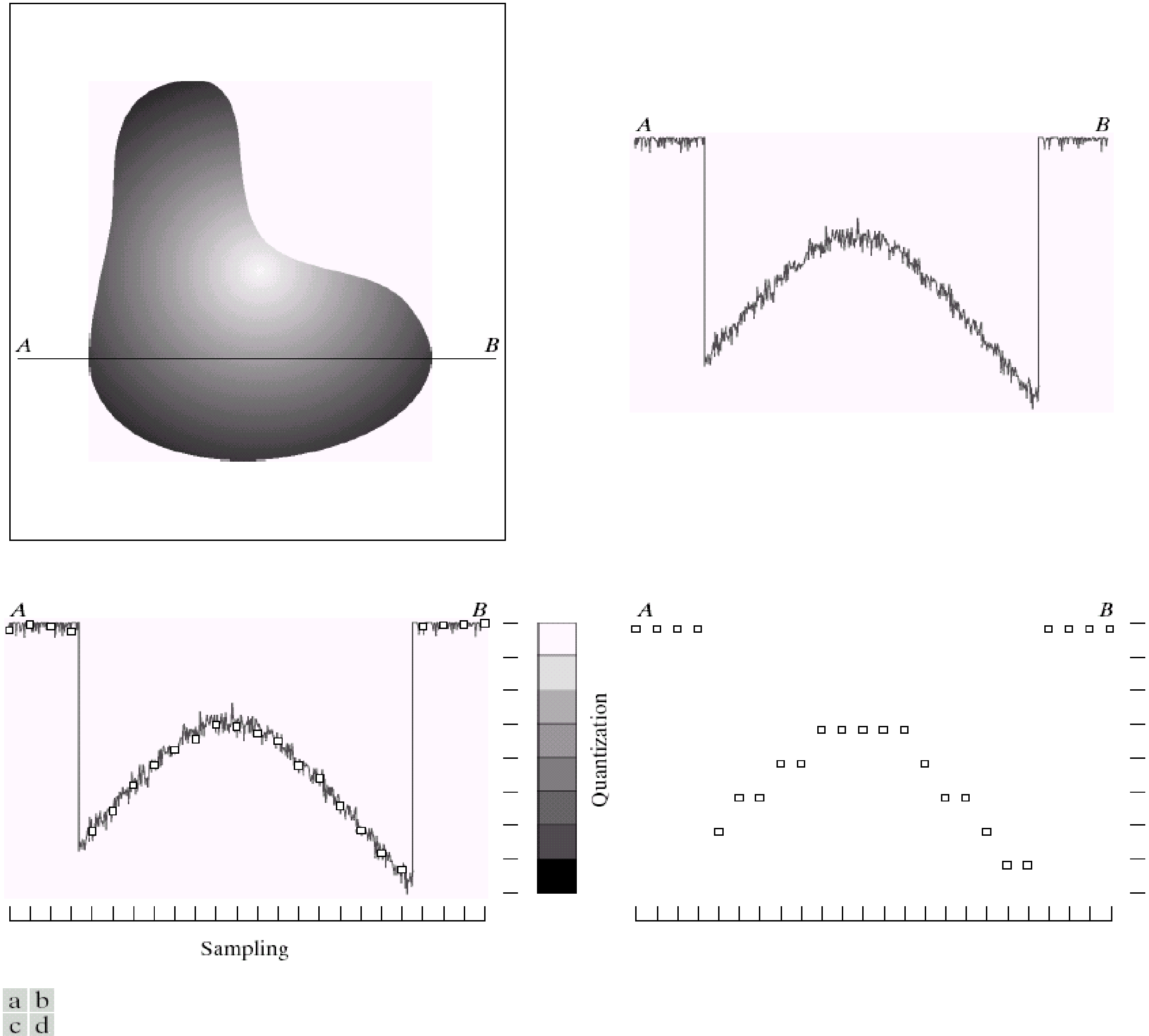
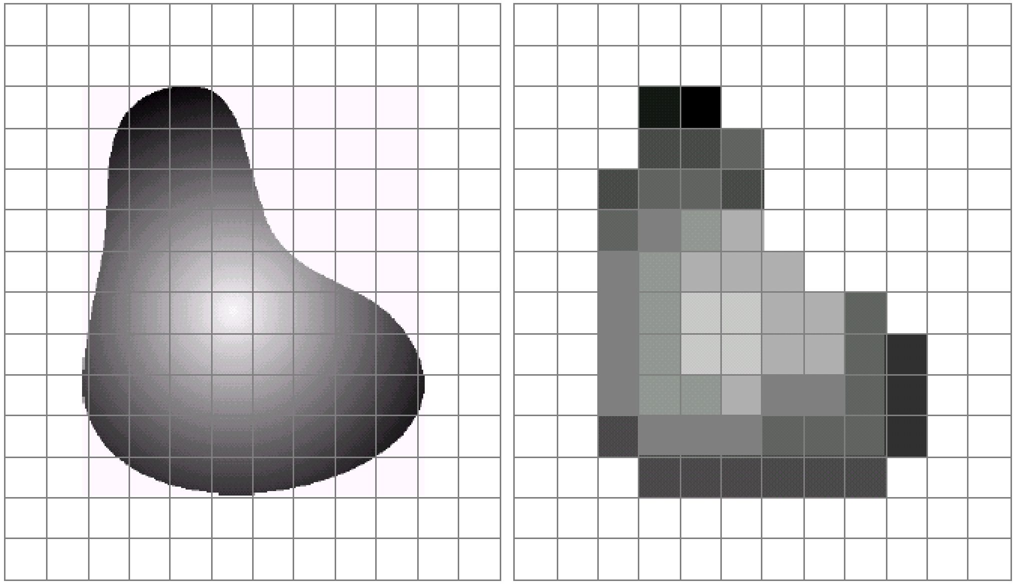


FIGURE 2.16 Generating a digital image. (a) Continuous image. (b) A scan line from A to B in the continuous image, used to illustrate the concepts of sampling and quantization. (c) Sampling and quantization. (d) Digital scan line.

Spatial and Gray-level Resolution



a b

FIGURE 2.17 (a) Continuos image projected onto a sensor array. (b) Result of image sampling and quantization.

Spatial resolution: dimension of digitized image (e.g., VGA 640-by-480)
Gray-level resolution: how many different gray levels are there in the image? (e.g., 8-bits means 256 different levels)

I only cover basic concepts here. For more details, you can read Sec. 2.3-2.4 of the textbook

8-bit Grayscale Images

W=256

H=256

row

156	159	158	155 …
160	154	157	158 …
156	159	158	155 …
160	154	157	158 …
…	…	…	…

column

0 255

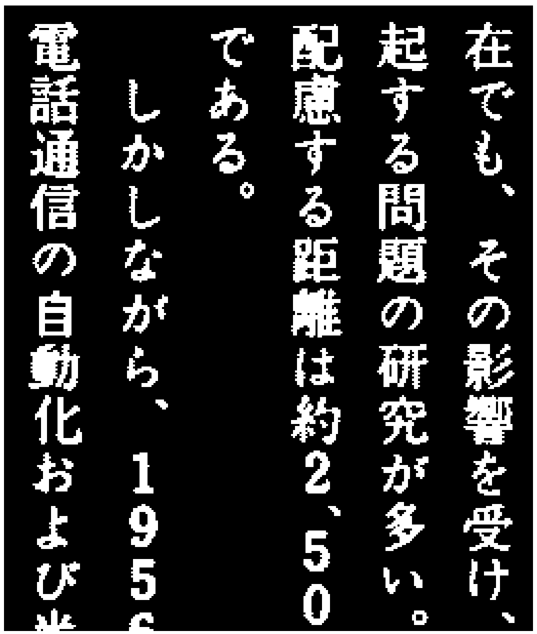
gray-level: 8 bits or 1 byte per pixel

Raw image data

No header information, just pack the pixel values in a rastering scanning order

The size of a raw image file is HW bytes (H,W are the height and width of the image)

Binary Images



0: black pixels

255: white pixels
(sometimes it is normalized to 1)

Color Images

Red

Green

Blue

Portable Grayscale Map (PGM), Portable Pixel Map (PPM) and Portable Bit Map (PBM)

Header Structure

P3 # magic number
1024 # the image width
788 # the image height
A comment
1023 # maximum value

: annotation

Magic number

P1: for PBM image

P2/P5: for PGM image (ASCII and binary respectively)

P3/P6: for PPM image (ASCII and binary respectively)

PGM/PPM/PBM Examples

PGM(8 bits)

header

```
P2
24 7
15
```

image data

```
0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
0  3  3  3  3  0  0  7  7  7  7  0  0 11 11 11 11  0  0 15 15 15 15  0
0  3  0  0  0  0  0  7  0  0  0  0  0 11  0  0  0  0  0 15  0  0 15  0
0  3  3  3  0  0  0  7  7  7  0  0  0 11 11 11  0  0  0 15 15 15 15  0
0  3  0  0  0  0  0  7  0  0  0  0  0 11  0  0  0  0  0 15  0  0  0  0
0  3  0  0  0  0  0  7  7  7  7  0  0 11 11 11 11  0  0 15  0  0  0  0
0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0  0
```

PPM(24 bits)

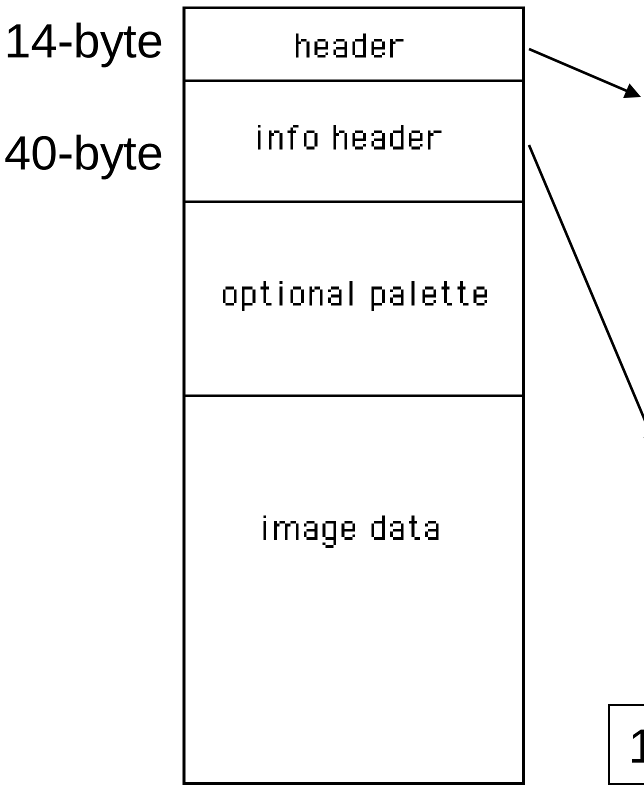
```
P3
# example from the man page
4 4
15
 0  0  0    0  0  0    0  0  0   15  0 15
 0  0  0    0 15  7    0  0  0    0  0  0
 0  0  0    0  0  0    0 15  7    0  0  0
15  0 15    0  0  0    0  0  0    0  0  0
```

PBM(1 bit)

```
P1
# PBM example
24 7
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
0 1 1 1 1 0 0 1 1 1 1 0 0 1 1 1 1 0 0 1 1 1 1 0
0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 1 0
0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 1 0 0 0 1 1 1 1 0
0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0 0 1 0 0 0 0
0 1 0 0 0 0 0 1 1 1 1 0 0 1 1 1 1 0 0 1 0 0 0 0
0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0
```

Review: Image Basics

Bitmap (BMP) Format

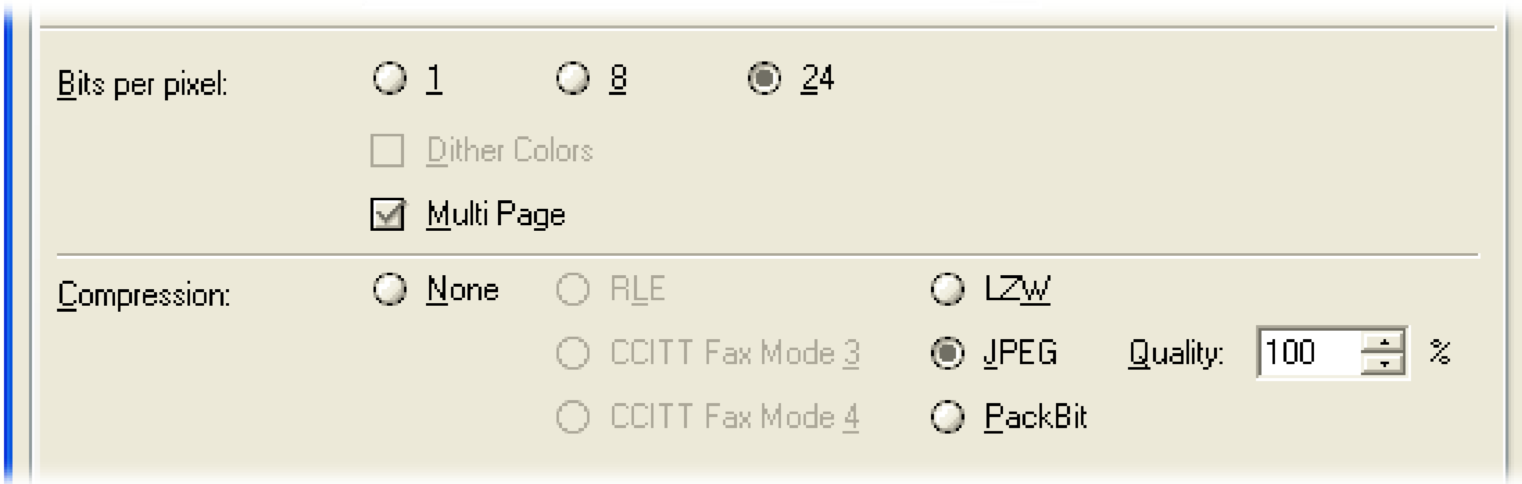



```
typedef struct {
   unsigned short int type;                 /* Magic identifier            */
   unsigned int size;                       /* File size in bytes          */
   unsigned short int reserved1, reserved2;
   unsigned int offset;                     /* Offset to image data, bytes */
} HEADER;
```

```
typedef struct {
   unsigned int size;                /* Header size in bytes      */
   int width,height;                 /* Width and height of image */
   unsigned short int planes;        /* Number of colour planes   */
   unsigned short int bits;          /* Bits per pixel            */
   unsigned int compression;         /* Compression type          */
   unsigned int imagesize;           /* Image size in bytes       */
   int xresolution,yresolution;      /* Pixels per meter          */
   unsigned int ncolours;            /* Number of colours         */
   unsigned int importantcolours;    /* Important colours         */
} INFOHEADER;
```

14 + 40 + 4 * infoheader.ncolours = header.offset

Tagged Image File Format (TIF/TIFF)



TIFF is a flexible bitmap image format supported by virtually all paint, image-editing, and page-layout applications. Also, virtually all desktop scanners can produce TIFF images.

Other Popular Image Formats

- GIF: Graphics Interchange Format
- PNG: Portable Network Graphics
- RAS: a format native to Sun UNIX platforms
- SGI: developed at Silicon Graphics, and is used for black and white, grayscale and color images.
- PCX: originally designed by ZSoft to be used by PC Paintbrush for MS-DOS. Microsoft later acquired the right to use the PCX format for Microsoft Paintbrush for Windows indirectly increasing the format's popularity.